

FORECASTING MINIMUM TEMPERATURES FOR THE CRANBERRY BOGS OF NEW JERSEY.

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[Weather Bureau Office, Philadelphia, Pa., September 19, 1922.]

For many years it has been known that cranberry bogs are very susceptible to late frosts in spring and early frosts in autumn, thus making the growing season much shorter than it is for the surrounding farm lands, and correspondingly increasing the danger to the cranberry crop. However, no definite move was made to study the conditions until the fall of 1905, when special cooperative stations were established on the bogs of Wisconsin, New Jersey, and Massachusetts.

Prof. Henry J. Cox studied the records from the Wisconsin bogs and soon concluded that the stations would need to be manned by trained observers if thoroughly accurate and reliable results were to be assured. Accordingly he assigned assistants from the Chicago station to the bogs in Wisconsin, and spent much of the time during the seasons of 1906 and 1907 in personal supervision of the work. He studied the matter intensively, and determined the values of the several factors, air drainage, radiation, soil temperatures, etc., which combine to produce the temperature conditions that obtain in the Wisconsin bogs. He also correlated these conditions with the weather maps, and gained a vast fund of information for use in forecasting.

In New Jersey the work was continued on a cooperative basis at Whitesbog, near New Lisbon, and in 1909 Professor Garriott made a somewhat cursory study and report of the results.

In the fall of 1917, Mr. C. A. Donnel spent the frost season in the New Jersey bogs, mostly at Whitesbog. He made a careful study of the physical conditions, and his report dealt chiefly with that phase of the problem. The following spring, Mr. J. B. Kincer visited Whitesbog and installed six minimum thermometers and furnished psychrometers for the use of the cooperative observer. The present system of telegraphing reports from the bogs during the frost season was adopted at that time. The temperature records obtained did not add materially to the known meteorological facts.

The writer took the frost records of the Whitesbog station and made a series of map studies for the purpose of correlating conditions as Professor Cox had done for the Wisconsin bogs. These studies revealed the fact that frost may occur on the bogs late in spring or early in autumn with minimum temperatures somewhat higher at surrounding points than are required earlier in spring or later in autumn. For example, during the first half of May frost rarely occurs at Whitesbog until the minimum temperature at Philadelphia is 50° or lower, but early in June there is danger of frost with a Philadelphia minimum of 56° if the radiation conditions are especially good. The reason for this has not been positively determined, but in spring it is believed to be due to the increasing density of the mat of leaves on top of the cranberry vines, which reduces absorption of heat by the soil during the daytime, and also makes a better radiation surface at night. In autumn the reverse process takes place to a certain extent as the leaves die and fall from the vines.

Another condition observed frequently in the map studies was that for a cool period of two or three days

duration the lowest bog temperature often followed at a time when a change to rising temperatures had begun in the surrounding country, and the weather map seemed to indicate that the worst was over.

In 1915 Prof. J. Warren Smith became interested in the forecasting of minimum temperatures by means of hygrometric formulæ, which he developed for orchard work at several of the Ohio stations. Later he believed that such formulæ could be used in forecasting for the cranberry bogs with possibly even greater success than obtained in orchard work, since radiation is a more important factor in bog conditions. Research work was reduced to a minimum during and for some time after the war, but in the autumn of 1921 he arranged with the Chief of Bureau for the beginning of a series of observations at Whitesbog, N. J. Mr. Clemmy C. Hamme was assigned as observer to work under the supervision of the writer.

Three locations were selected for the exposure of the instruments, station No. 1 being located on a dike, well out toward the center of a large bog area. The instrument shelter was raised 3 feet above the dike, so that the contained instruments were exposed about 8 feet above the bog, and beyond the immediate effects of radiation. An anemometer was exposed at this station 12 feet above the bog. Station No. 2 was located about a thousand feet from station No. 1, and near the center of the lowest, and consequently the coldest, of all the bogs. Here a minimum thermometer and a 29-hour thermograph were exposed, in the open, about 3 or 4 inches above the top of the vines. Station No. 3 was placed in the woods about 500 feet from the northwestern border of the bog.

Observations were taken from September 21 to October 17, inclusive, under somewhat unfavorable conditions, as radiation was poor the greater portion of the time. There were 10 good, or fairly good, radiation nights. The work was resumed the following spring, observations being taken from May 6 to June 19, inclusive, under more favorable conditions, there being 15 good, or fairly good, radiation nights. The spring series of observations was made by Mr. Charles I. Dague, observer, of the Weather Bureau.

A practical working formula can not be determined from ideal radiation nights only, but must include all conditions which give the grower much concern. Fifteen nights can be selected from the two series of observations which give a formula that is wonderfully accurate when applied to ideal radiation conditions only, but it is not a satisfactory formula for general use. On the 25 dates above mentioned the bog minimum was below 40° F. in each instance, and was considerably below either the dewpoint or the shelter minimum at station No. 1. Some of the nights were partly overcast with high clouds, but each was of such a character as to give a frosty feeling to the air during the early part of the night and thus cause some uneasiness on the part of the grower. Table 1 gives the data which finally determined the selection of the nights mentioned.

TABLE 1.

Night.	8:00 p. m.		8:00 a. m.		Depression of bog minimum.	
	Relative humidity.	Dew-point.	Shelter minimum.	Bog minimum.	Below dewpoint.	Below shelter minimum.
1921.						
Sept. 26-27.....	92	47	41.0	29.8	17.2	11.2
30-Oct. 1.....	92	55	40.0	29.0	26.0	11.0
Oct. 1-2.....	84	45	40.0	29.3	15.7	10.7
4-5.....	85	40	41.0	29.4	10.6	11.6
6-7.....	76	45	38.0	29.8	15.2	8.2
8-9.....	80	38	31.4	22.0	16.0	9.6
10-11.....	83	52	39.0	31.9	20.1	7.1
12-13.....	77	37	29.8	22.0	15.0	7.8
13-14.....	89	32	27.7	18.9	13.1	8.8
14-15.....	96	39	28.8	21.4	17.6	7.4
1922.						
May 8-9.....	26	27	40.6	31.0	4.0	9.6
10-11.....	65	54	51.2	39.5	14.5	11.7
12-13.....	71	43	36.8	28.7	14.3	8.1
15-16.....	78	52	44.4	33.7	18.3	10.7
16-17.....	77	45	41.8	31.3	13.7	10.5
19-20.....	50	44	46.9	38.5	5.5	8.4
23-24.....	82	54	45.5	38.5	15.5	7.0
24-25.....	60	46	46.4	35.8	10.2	10.6
27-28.....	82	45	38.6	28.8	16.2	9.8
28-29.....	64	42	40.5	30.3	11.7	10.2
29-30.....	55	49	47.0	34.5	14.5	12.5
30-31.....	78	55	47.1	37.2	17.8	9.9
June 12-13.....	38	39	45.3	33.0	6.0	12.3
13-14.....	81	48	50.9	39.4	8.6	11.5
14-15.....	73	52	44.6	35.1	16.9	9.5

The median hour was determined for these nights, but gave very erratic and unsatisfactory results when used for computing the minimum temperatures. The straight line formula ($Y=a+bR$) determined by the method of least squares, gives good results in most instances, as shown by Table 2. In this table Y is the depression of the bog minimum below the 8:00 p. m. dewpoint, and R is the 8:00 p. m. relative humidity.

TABLE 2.

Night.	R	Y	R^2	RY	Bog minimum.	Computed minimum.	Error.
1921.							
Sept. 26-27.....	92	-17	8,464	-1,564	29.8	28.1	-1.7
30-Oct. 1.....	92	-26	8,464	-2,392	29.0	36.1	+7.1
Oct. 1-2.....	84	-16	7,056	-1,344	29.3	28.2	-1.1
4-5.....	85	-11	7,225	-935	29.4	23.0	-6.4
6-7.....	76	-15	5,776	-1,140	29.8	30.4	+0.6
8-9.....	80	-16	6,400	-1,280	22.0	22.3	+0.3
10-11.....	83	-20	6,889	-1,660	31.9	35.5	+3.6
12-13.....	77	-15	5,929	-1,155	22.0	22.1	+0.1
13-14.....	89	-13	7,921	-1,157	18.9	13.9	-5.0
14-15.....	96	-18	9,216	-1,728	21.4	19.0	-2.4
1922.							
May 8-9.....	26	+4	676	+104	31.0	25.9	-5.1
10-11.....	65	-14	4,225	-910	39.5	42.4	+2.9
12-13.....	71	-14	5,041	-994	28.7	29.8	+1.1
15-16.....	78	-18	6,084	-1,404	33.7	36.9	+3.2
16-17.....	77	-14	5,929	-1,078	31.3	30.1	-1.2
19-20.....	50	-6	2,500	-300	38.5	36.4	-2.1
23-24.....	82	-16	6,724	-1,312	38.5	37.8	-0.7
24-25.....	60	-10	3,600	-600	35.8	35.7	-0.1
27-28.....	82	-16	6,724	-1,312	28.8	28.8	0.0
28-29.....	64	-12	4,096	-768	30.3	30.6	+0.3
29-30.....	55	-14	3,025	-770	34.5	40.1	+5.6
30-31.....	78	-18	6,084	-1,404	37.2	39.9	+2.7
June 12-13.....	38	-6	1,444	-228	33.0	34.7	+1.7
13-14.....	81	-9	6,561	-729	39.4	32.1	-7.3
14-15.....	73	-17	5,329	-1,241	35.1	38.2	+3.1
Sum.....	1834	-347	141,372	-27,291			

$$b = -0.27$$

$$a = 5.93$$

On the night of September 30-October 1 there was seemingly a strong anticyclonic cooling in addition to highly favorable radiation conditions. It was an un-

usual condition that was not indicated by the weather map. Cloudiness increased during the nights of October 4-5 and June 13-14, lessening radiation so that the temperature did not fall so low as the formula indicated. However, in most instances the results are as accurate as it is possible to determine the dewpoint by means of the ordinary whirling psychrometer.

Following the spring series of observations, Mr. Dague returned to Washington and worked out some very interesting results, based chiefly on the records he had made. The following is quoted from his report:

"Several methods of forecasting the minimum temperatures from the available hygrometric data for radiation nights have been considered. The most successful method is a slight modification of the formula used at Pomona, Calif., by Mr. Floyd D. Young. This formula, as modified, is given below with the corrections to be applied for the varying values of the dewpoint and relative humidity. T , minimum temperature indicated for the following morning; D , dewpoint; and H , relative humidity at the evening observation; V , variable quantity depending upon the temperature of the dewpoint; V' , variable quantity depending upon the relative humidity.

Weather at time of observation.

$$\text{Clear or partly cloudy, } T = D - \frac{H-25}{4} + V + V'.$$

$$\text{Cloudy, } T = D - \frac{H-30}{4} + V + V'.$$

Dewpoint ($^{\circ}$ F):

V	Relative humidity (per cent):	V'
27.....	55 to 59.....	-1
28.....	60 to 64.....	-2
29 to 30.....	65 to 70.....	-3
31 to 33.....	71 to 72.....	-4
34 to 35.....	73 to 83.....	-5
36 to 37.....	89 to 93.....	-6
38 to 39.....		
40 to 41.....		
42.....		
43 to 45.....		
46 to 47.....		
48.....		
49 to 50.....		
51.....		

"The application of this modified formula to new data for Whitesbog, N. J., May-June, 1922, for all of the dates with a bog minimum below 40° is given in Table 3 for the 8:00 p. m. observations and in Table 4 for the noon observations.

TABLE 3.—Observations taken at 8:00 p. m.

Date.	Dew-point.	Relative humidity.	Weather, 8:00 p. m.	Forecast minimum.	Actual minimum.	Error.
	$^{\circ}$ F.	P. cent.		$^{\circ}$ F.	$^{\circ}$ F.	$^{\circ}$ F.
May 8.....	27	26	Cloudy.....	31	31	0
12.....	43	71	Clear.....	30	29	+1
15.....	52	78	do.....	34	34	0
16.....	45	77	do.....	31	31	0
19.....	44	50	do.....	32	38	-6
23.....	44	82	do.....	35	38	-3
24.....	46	60	do.....	32	36	-4
27.....	45	82	Pt. cldy.....	30	29	+1
28.....	42	64	Clear.....	29	30	-1
29.....	49	55	do.....	33	34	-1
30.....	55	78	do.....	37	37	0
June 12.....	39	38	do.....	33	33	0
13.....	48	81	Pt. cldy.....	31	39	-8
14.....	52	73	Cloudy.....	35	35	0
18.....	68	89	Pt. cldy.....	48	49	-1

¹ Sky entirely overcast by 8:45 p. m.

TABLE 4.—Observations taken at 12:00 noon.

Date.	Dew-point.	Relative humidity.	Weather, 12:00 noon.	Forecast minimum.	Actual minimum.	Error.
May 8.....	38	33	Clear.....	29	31	-2
12.....	29	20	Pt. cldy.....	27	29	-2
15.....	50	42	Cloudy.....	34	34	0
16.....	43	32	do.....	31	31	0
19.....	54	48	Pt. cldy.....	34	38	-4
23.....	55	43	do.....	37	38	-1
24.....	54	39	Clear.....	36	36	0
27.....	48	71	Cloudy.....	31	29	+2
28.....	45	43	Pt. cldy.....	31	30	+1
29.....	52	37	Clear.....	35	34	+1
30.....	53	37	do.....	36	37	-1
June 12.....	48	36	do.....	33	33	0
13.....	36	31	Cloudy.....	34	1 39	-5
14.....	48	45	do.....	30	35	-5
18.....	72	81	Pt. cldy.....	49	49	0

¹ Sky entirely overcast by 8:45 p. m.

"In Table 3 the largest error in the forecast minimum temperature, -8° , was due to a rapid clouding up shortly after the evening observation, the sky remaining overcast the remainder of the night. The next largest, -6° , was probably due to the wet condition of the bog, as it had recently been flooded for protection against frost.

"In applying the formula to the noon observations it was necessary to make a change in the dewpoint factors by increasing each factor 4° in order to obtain the forecast minimum temperatures as given in Table 4. With one or two exceptions the minimum temperatures which were forecast show a good agreement with the actual bog minimum, and, as in Table 3, the larger departures are on the safe side.

"Further application of the formula was made to hygrometric data at the regular Weather Bureau station at Philadelphia, Pa., for both the 12:00 noon and the 8:00 p. m. observations, and correlated with the ensuing minimum temperatures at Whitesbog, which is about 40 miles northeast from Philadelphia. Some changes in the formula were necessary before it could be applied to the Philadelphia data. The relative humidity factors were omitted for both the 12:00 noon and 8:00 p. m. observations. For the 12:00 noon observations each of the dewpoint factors was increased 2° . The results are given in Tables 5 and 6.

TABLE 5.—Application of hygrometric formula, using hygrometric data at Philadelphia, Pa., and bog minimum temperatures at Whitesbog (New Lisbon), N. J. Observations taken at 8:00 p. m.

Date.	Dew-point.	Relative humidity.	Weather, 8:00 p. m.	Forecast minimum.	Actual minimum.	Error.
	$^{\circ}$ F.	P. cent.		$^{\circ}$ F.	$^{\circ}$ F.	$^{\circ}$ F.
May 8.....	29	23	Clear.....	30	31	-1
12.....	39	45	do.....	31	29	+2
15.....	55	71	Pt. cldy.....	34	34	0
16.....	42	48	Clear.....	31	31	0
19.....	51	54	do.....	34	38	-4
23.....	49	48	do.....	34	38	-4
24.....	42	38	do.....	31	36	-5
27.....	45	71	do.....	27	29	-2
28.....	49	60	do.....	31	30	+1
29.....	52	48	do.....	36	34	+2
30.....	54	58	do.....	36	37	-1
June 12.....	43	36	do.....	34	33	+1
13.....	40	35	Cloudy.....	35	1 39	-4
14.....	46	48	do.....	35	35	0
18.....	66	74	Pt. cldy.....	49	49	0

¹ Sky entirely overcast by 8:45 p. m.

TABLE 6.—Application of hygrometric formula, using hygrometric data at Philadelphia, Pa., and bog minimum temperatures at Whitesbog (New Lisbon), N. J. Observations taken at 12:00 noon.

Date.	Dew-point.	Relative humidity.	Weather, 12:00 noon.	Forecast minimum.	Actual minimum.	Error.
May 8.....	37	34	Clear.....	31	31	0
12.....	29	23	Cloudy.....	30	29	+1
15.....	51	50	do.....	34	34	0
16.....	40	29	Clear.....	33	31	+2
19.....	49	48	Pt. cldy.....	32	38	-6
23.....	55	49	do.....	37	38	-1
24.....	52	41	Clear.....	36	36	0
27.....	51	73	Cloudy.....	28	29	-1
28.....	45	45	Clear.....	31	30	+1
29.....	45	31	do.....	35	34	+1
30.....	44	30	do.....	35	37	-2
June 12.....	43	28	do.....	34	33	+1
13.....	34	31	Pt. cldy.....	30	1 39	-9
14.....	54	60	Clear.....	34	35	-1
18.....	67	69	Cloudy.....	45	49	-4

¹ Sky entirely overcast by 8:45 p. m.

"Dot charts were made from the data obtained at Whitesbog during May and June 1922. The relative humidity data, as determined by the 12:00 noon and 8:00 p. m. observations of the dry and wet bulb temperatures, are indicated on the bottom of the diagrams, while the figures entered at the left are the differences between the dewpoint temperature at the noon or evening observations and the ensuing bog minimum temperatures.

"A dot is entered on the chart for each day to agree with the observed relative humidity and the variation of the bog minimum from the dewpoint temperature. The arrangement of the dots on both figures indicates that a parabolic curve is probably the line of best fit. These parabolic curves ($v = x + by + cz$) have been calculated for both charts and the curves drawn. In Figure 2, however, it seems that a straight line would probably fit the arrangement of the dots about as well as the parabolic curve, so the straight line ($y = a + bR$) has also been calculated and superimposed upon this chart. The coefficients from these equations for the lines are indicated on the respective figures.

"The arrangements of the dots about these lines indicates a close relationship between the relative humidity and the variation of the ensuing bog minimum from the dewpoint temperature. It is believed that an even closer relationship will be shown with a larger number of observations.

"A detailed examination of the dewpoint temperatures indicates that a bog minimum temperature low enough to damage the buds on the vines in spring is hardly probable when the dewpoint at the evening observation is above 45° .

"A 29-hour thermograph was exposed at each of the three stations, and a comparison of the traces shows that temperature conditions near the top of the vines change in advance of those of the layers of air higher up; that the minimum is reached sooner near the vines (on the average an hour or an hour and a half in advance); on some nights the change being as much as three or four hours in advance.

"From the available records at the Central Office, all bog minimum temperatures under 40° F. occurring at Whitesbog during the spring and fall frost season of 1919, 1920, 1921, and 1922 were selected for correlation

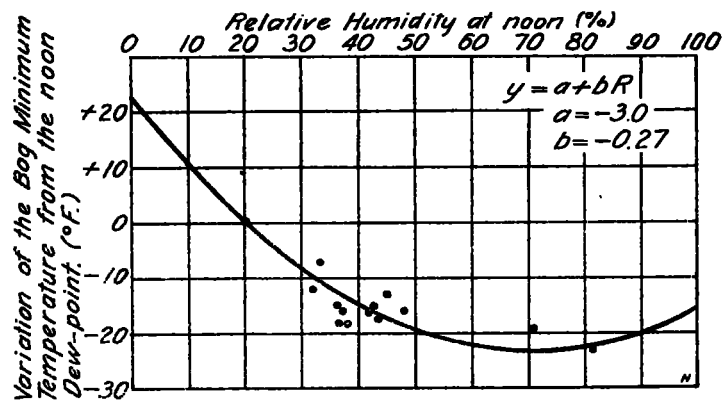


FIG. 1.—Relation between the 12 o'clock noon relative humidity and the variation of the bog minimum temperature during the night from the noon dewpoint. Whitesbog, N. J., May-June, 1922.

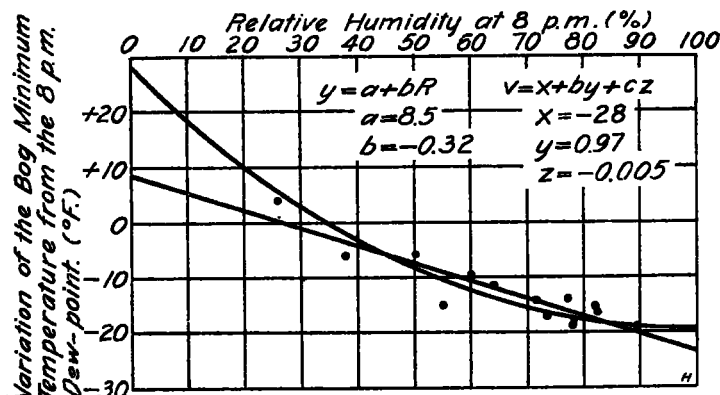


FIG. 2.—Relation between the 8 p. m. relative humidity and the variation of the bog minimum temperature during the night from the 8 p. m. dewpoint. Whitesbog, N. J., May-June, 1922.

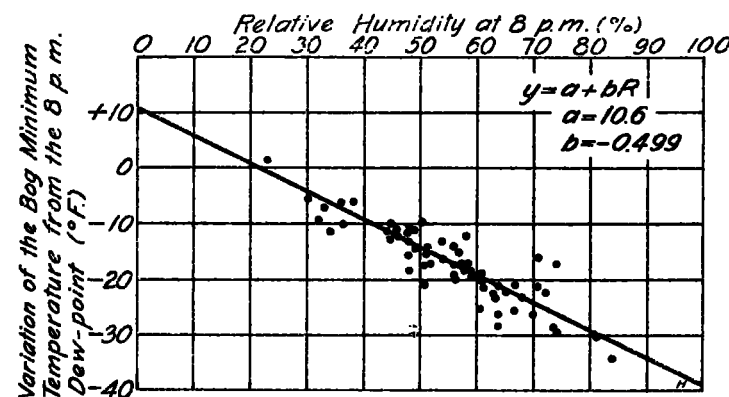


FIG. 3.—Relation between 8 p. m. relative humidity at Philadelphia, Pa., and variation of bog minimum temperature during the night at Whitesbog, N. J., from the 8 p. m. dewpoint at Philadelphia. May, June, September, and October, 1919, 1920, 1921, and 1922.

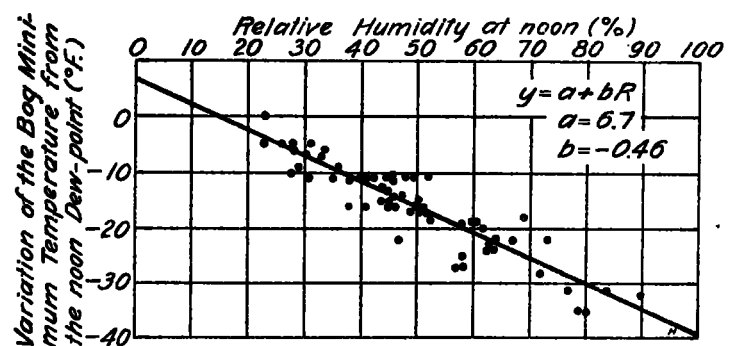


FIG. 4.—Relation between 12 o'clock noon relative humidity at Philadelphia, Pa., and variation of bog minimum temperature during the night at Whitesbog, N. J., from the 12 o'clock noon dewpoint at Philadelphia. May, June, September, and October, 1919, 1920, 1921, and 1922.

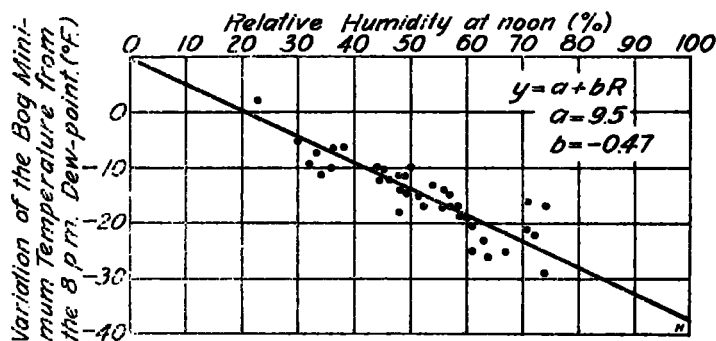


FIG. 5.—Relation between 8 p. m. relative humidity at Philadelphia, Pa., and variation of bog minimum temperature during the night at Whitesbog, N. J., from the 8 p. m. dewpoint at Philadelphia. May and June, 1919, 1920, 1921, and 1922.

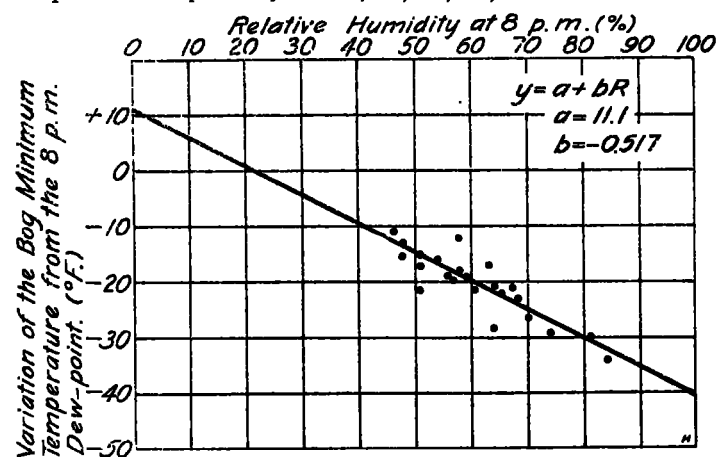


FIG. 6.—Relation between 8 p. m. relative humidity at Philadelphia, Pa., and variation of bog minimum temperature during the night at Whitesbog, N. J., from the 8 p. m. dewpoint at Philadelphia. September and October, 1919, 1920, 1921, and 1922.

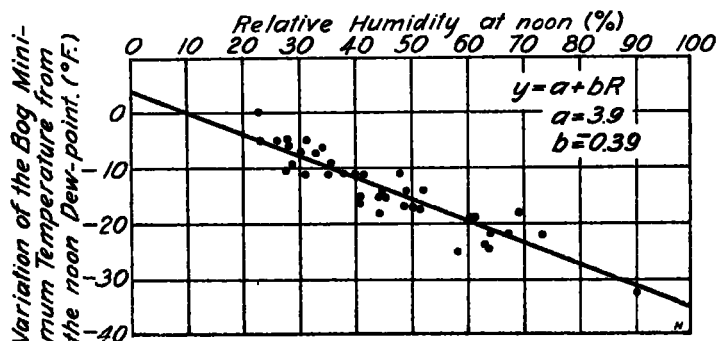


FIG. 7.—Relation between 12 o'clock noon relative humidity at Philadelphia, Pa., and variation of the bog minimum temperature during the night at Whitesbog, N. J., from the 12 o'clock noon dewpoint at Philadelphia. May and June, 1919, 1920, 1921, and 1922.

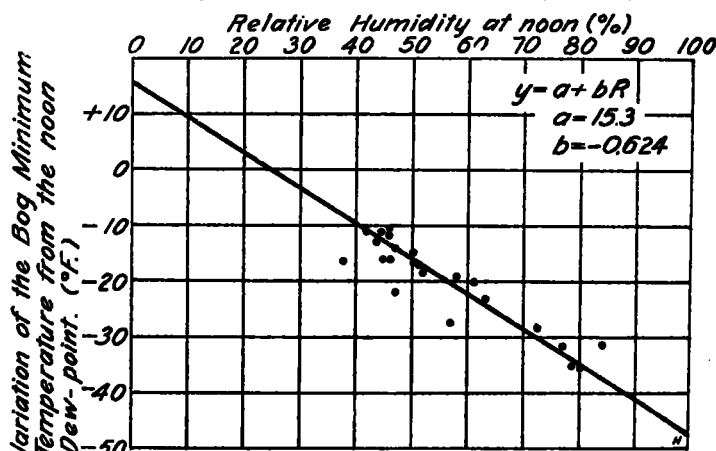


FIG. 8.—Relation between 12 o'clock noon relative humidity at Philadelphia, Pa., and variation of the bog minimum temperature during the following night at Whitesbog, N. J., from the 12 o'clock noon dewpoint at Philadelphia. September and October, 1919, 1920, 1921, and 1922.

with hygrometric data obtained at the regular observations of the Weather Bureau Office at Philadelphia. By means of the barograph and thermograph trace sheets from Philadelphia and the corresponding weather maps, the bog minimum temperatures on those days which clearly showed that they were not of radiation origin were eliminated. Thus the writer could feel quite sure he was using data representative, or fairly so, of radiation conditions. In all, 61 observations were used.

"Dot charts were made, using bog minimum temperature data for Whitesbog and hygrometric data for Philadelphia, following the same plan for plotting these data as was used in Figures 1 and 2. These charts are shown in Figures 3 to 8, inclusive.

"From the arrangement of the dots on each chart, a straight line appears to be the best fit and the equation ($y=a+bR$) has been calculated, the coefficients of which have been entered on the respective charts.

TABLE 7.—Distribution of departures of forecast bog minimum temperatures made from hygrometric data at Philadelphia, Pa., from actual bog minimum temperatures at Whitesbog (New Lisbon) N. J., using Figures 3 to 8, inclusive.

Departures.	Fig. 3.	Fig. 4.	Fig. 5.	Fig. 6.	Figs. 5 and 6 combined.	Fig. 7.	Fig. 8.	Figs. 7 and 8 combined.
0.....	16	9	8	4	12	5	5	10
±1.....	9	22	6	12	18	17	5	22
±2.....	16	10	8	3	11	6	7	13
±3.....	7	7	10	0	10	4	2	6
±4.....	4	4	1	0	1	3	0	3
±5.....	4	5	3	1	4	2	0	2
±6.....	2	0	1	2	3	1	2	3
±7.....	1	3	0	1	1	0	1	1
±8.....	0	1	1	0	1	0	1	1
±9.....	2	0	0	0	0	0	0	0
0.....	16	9	8	4	12	5	5	10
+	22	25	13	9	22	14	8	22
-	23	27	17	10	27	19	10	29
Total.....	61	61	38	23	61	38	23	61

"Figures 3 and 4 show the data charted for the total number of 12:00 noon and 8:00 p. m. observations for both the spring and fall seasons. Figures 5 to 8, inclusive,

were charted to ascertain if the accuracy of the bog minimum temperatures forecast could be increased if the spring and fall seasons were separately considered for both the noon and evening data. The accuracy was slightly increased but not so much as was hoped for, although some of the largest departures were eliminated. Table 7 gives in summarized form the remarkable results obtained.

"The data in Table 7 show that the forecast bog minimum temperatures for Whitesbog, based on hygrometric data at Philadelphia, were within 2° F. of the actual bog minimum temperatures from 65 to 75 per cent of the time, while departures in excess of 4° were very few indeed."

CONCLUSION.

(1) In conclusion it may be said that for the best possible results in forecasting minimum temperatures for cranberry bogs a good hygrometric formula is necessary for use with the weather map. (2) That when the weather map indicates the radiation conditions to be good, the hygrometric formula will give a closer and more uniformly consistent estimate of the probable bog minimum temperatures than it is possible to obtain otherwise. (3) That a hygrometric formula, correlated between the bog minima and data from a near-by Weather Bureau station, will give fairly accurate and reliable results, and is a valuable aid to the forecaster. (4) That a hygrometric formula adapted to a given locality, if intelligently and accurately applied by the grower, would be a fairly reliable safeguard without other information. (5) That in the latter case, when general radiations were not so good as they appeared to be locally, the formula might give too low a temperature and cause the grower to flood the bogs unnecessarily and at some loss, which the aid of the weather map would in most cases avoid. (6) That under ideal radiation conditions the error of a well-calculated hygrometric formula seems to be no larger than the personal equation in obtaining the data. (7) That the data for at least 50 good, or fairly good, radiation nights should be used in calculating the formula.

A SIMPLE GEOMETRIC DERIVATION OF THE LAWS OF REFRACTION OF LIGHT INCLINED TO A PRINCIPAL PLANE OF A PRISM.

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In the theory of halos, as also in any general discussion of the action of refracting prisms on light, it is necessary to consider the course of rays inclined at various angles to a principal plane—that is, a plane normal to both the face through which the light enters the prism and the face through which it leaves the prism, or, what comes to the same thing, normal to the intersection of these two faces.

This problem was first discussed fully by the French astronomer Auguste Bravais, and the laws found (often called Bravais' laws of refraction) used in his classical memoir on halos.¹ They have also been variously derived by other writers, most recently by Terpstra,² and, with mathematical elegance and dispatch, by Laville.³ However, none, presumably, of these various derivations, some of which are tedious to follow, is readily available to the average reader of the REVIEW. It may be worth while, therefore, to give those naturally most interested

in halos a straightforward discussion of the refraction of light rays inclined to a principal plane.

In what follows it will be necessary to remember the well-known facts:

1. The angle of incidence is the angle between the incident ray and the normal to the surface at the point of incidence.

2. The angle of refraction is the angle between the refracted ray and the normal (within the prism) to the surface at the point of refraction.

3. The plane of the incident and refracted rays is normal to the incident surface at the point of incidence. Similarly, the plane of the refracted and exit rays is normal to the exit surface at the point of exit.

4. If i is the angle of incidence and r the angle of refraction, then $\sin i = \mu \sin r$, where μ , called the index of refraction, is the ratio of the velocity of light in the surrounding medium, air, say, to its velocity within the prism.

Remembering these facts, let ABC , Figure 1, be a principal plane of a prism; let DEF be the plane, perpen-

¹ Journal de l'Ecole Royale Polytechnique, 18, 1847.

² Zeit. f. den phys. und chem. Unterricht, p. 80, March, 1922.

³ Journal de Phys. et le Radium, 2, p. 62, 1921.